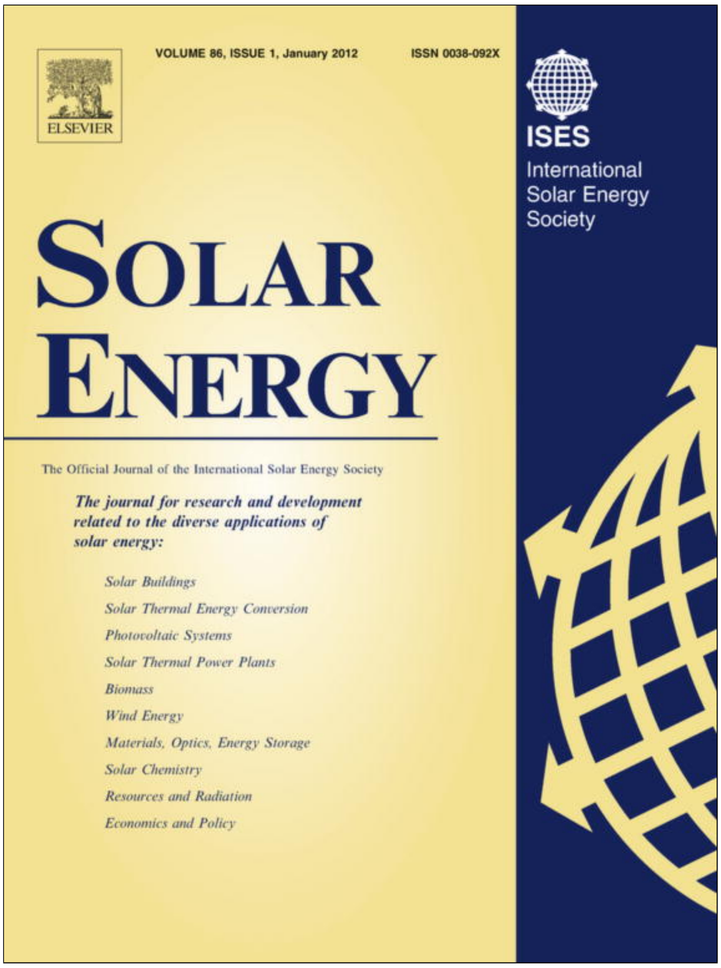
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Analytical derivation of explicit J–V model of a solar cell from physics based implicit model

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Abstract

Recently a simple explicit model was introduced to represent the J–V characteristics of an illuminated solar cell with parasitic resistances and bias dependent photocurrent as $v^m + j^n = 1$. Here the normalized voltage, v and normalized current density j can be represented as $v = V/V_{oc}$ and $j = J/J_{sc}$ respectively, where V_{oc} is the open circuit voltage and J_{sc} is the short circuit current density. This model is useful for design, characterization and simple fill factor calculation and its applicability was demonstrated with the measured data of a wide variety of solar cells. This explicit form is intuitive and hence the model lacks the analytical support. In this paper an analytical derivation of this closed form explicit model is presented, which is derived from the physics based implicit J–V equation. The derivation expands the scope of model applicability and provides a new insight of analytical modeling of the solar cell.

Keywords: Analytical model; Solar cell; Explicit J–V model

1. Introduction

One of the best options for future sustainable energy requirements of the world is the electricity generated from solar cells (Razykov et al., 2011). For an illuminated solar cell having parasitic series and shunt resistances, the simplest of the current density–voltage (J–V) equations called the Single Exponential Model (SEM), has an implicit form

$$J = J_{ph} - J_0\left\{\exp\left(\frac{V + JR_s}{\eta V_t}\right) - 1\right\} - \frac{V + JR_s}{R_{sh}}, \tag{1}$$

where J_0 is the dark current density; J_{ph} is the photogenerated current density; V_t is the thermal voltage at temperature T; η is the ideality factor; R_s is the unit area parasitic series resistance; and R_{sh} is the unit area parasitic shunt resistance. Different expressions for J_{phv} have been proposed in literature (for a-Si (Hegedus, 1997; Hegedus and Philips, 1994) and a-SiGe cells (Hegedus, 1997), for BEH_1BMB_3–PPV:PCBM polymer cells (Mihailetchi et al., 2005) and MDMO–PPV:PCBM polymer cells (Mihailetchi et al., 2004), etc.).

The implicit form of (1) calls for iterative calculations to compute the maximum power point (V_{mpp}, J_{mpp}) and the fill-factor (FF) in terms of physical parameters (Zhu et al., 2011). For simplification of the calculation, efforts have been made to transform (1) to an explicit form (Karmalkar and Haneefa, 2008; Jain and Kapoor, 2004; Banwell and Jayakumar, 2000; Ortiz-Conde and Garcia Sanchez, 1992; Abuelma'atti, 1992; Fjeldly et al., 1991; Saloux et al., 2011; Das and Karmalkar, 2011). Some semi-empirical approach is also used on solar panel (similar expression like (1)) to estimate the J–V (De Soto et al., 2006).

Recently an explicit model was proposed in Das (2011), which is validated with wide variety of solar cells. Similar modeling approach is also introduced in Saetre et al. (2011). Denoting the short circuit current density as J_{sc}

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and the open circuit voltage as V_{oc}, the normalized voltage, v and normalized current density j can be represented as $v = V/V_{oc}$ and $j = J/J_{sc}$ respectively. This representation shows that the simple analytical explicit function fits the wide variety of J–V measurement accurately (Das, 2011; Saetre et al., 2011):

$$v^m + j^n = 1 \tag{2}$$

This analytical model (2) allows the closed form estimation of the entire J–V curve and is useful for a simple calculation of maximum power point and fill factor. But this equation has lacked analytical support as it has been arrived at empirically by intuition.

In this paper it is shown that, (1) can be transformed into (2) by series of algebraic manipulations. This paper provides an analytical basis for the proposed explicit model (2). The derivation is presented in Section 2.

2. Derivation

Assuming $J_0 \ll J_{ph}$, (1) transforms to

$$J \approx \frac{J_{phv}}{1 + R_s/R_{sh}} - \frac{V/R_{sh}}{1 + R_s/R_{sh}} - \left(\frac{J_0}{1 + R_s/R_{sh}}\right) \exp\left(\frac{V + JR_s}{\eta V_t}\right) \tag{3}$$

Set $J = J_{sc}$ for $V = 0$ to get

$$J_{sc} = \frac{J_{phsc}}{1 + R_s/R_{sh}}\left[1 - \left(\frac{J_0}{J_{ph}}\right) \exp\left(\frac{J_{sc}R_s}{\eta V_t}\right)\right] \approx \frac{J_{phsc}}{1 + R_s/R_{sh}} \tag{4}$$

Set $V = V_{oc}$ for $J = 0$ to get

$$J_{phoc} - J_0 \exp\left(\frac{V_{oc}}{\eta V_t}\right) - \frac{V_{oc}}{R_{sh}} = 0 \tag{5}$$

Rewrite (3) in a normalized form by dividing throughout by J_{sc} of (4), substituting for J_0/J_{phoc} from (5),

$$j \approx \frac{J_{phv}}{J_{phsc}} - \left(\frac{V_{oc}}{J_{phsc}R_{sh}}\right) v - \left(\frac{J_{phoc}}{J_{phsc}} - \frac{V_{oc}}{J_{phsc}R_{sh}}\right) \mathrm{X} \exp[\alpha_{sc} j - \alpha_{oc}(1 - v)] \tag{6}$$

where $v = V/V_{oc}$ and $j = J/J_{sc}$, $\alpha_{oc} = \frac{V_{oc}}{\eta V_T}$ and $\alpha_{sc} = \frac{J_{sc}R_s}{\eta V_T}$. For simplification of the calculation, let us consider $\alpha = \frac{V_{oc}}{J_{phsc}R_{sh}}$ and $\beta = \frac{J_{phoc}}{J_{phsc}}$; hence Eq. (6) transforms into

$$j = 1 - \alpha v - \varphi(v) - (\beta - \alpha) \exp\{\alpha_{oc}(v - 1) + \alpha_{sc} j\} \tag{7}$$

where $\varphi(v)$, which is due to bias dependent photocurrent, can be represented as $\varphi(v) = 1 - (J_{phv}/J_{phsc})$. Since $v < 1$, we can express the term $(v - 1)$ in terms of $\log v$ using Taylor's series expansion of $\log v = \log(1 - 1 - v)$ in the form of $(v - 1)\delta_v$ where

$$1 \leqslant \delta_v = 1 + \frac{(1 - v)}{2} + \frac{(1 - v)^2}{3} + \ldots \leqslant \infty \tag{8}$$

So that

$$v - 1 = \frac{\log v}{\delta_v} \tag{9}$$

Similarly, since $j < 1$, using Taylor's series expansion of $\log(1 - j)$, we can state that

$$j = \frac{-\log(1 - j)}{\delta_j} \tag{10}$$

where

$$1 \leqslant \delta_j = 1 + \frac{j}{2} + \frac{j^2}{3} + \ldots \leqslant \infty \tag{11}$$

Hence using (9) and (10), replacing $(v - 1)$ and j terms in the right hand terms of (7), we can state that

$$j = 1 - \alpha v - \varphi(v) - (\beta - \alpha) v^{\frac{\alpha_{oc}}{\delta_v}} (1 - j)^{-\frac{\alpha_{sc}}{\delta_j}} \tag{12}$$

The J–V characteristic of a solar cell is linear when v is very small and near to 0, and shows a non-linear characteristic when v is high and close to 1. When v is small (j is high) we can consider $j \approx 1 - \alpha v - \varphi(v)$, which represents the linear relationship between v and j. For algebraic simplification let us divide (12) by $1 - \alpha v - \varphi(v)$,

$$\frac{j}{1 - \alpha v - \varphi(v)} = 1 - \frac{(\beta - \alpha)}{1 - \alpha v - \varphi(v)} v^{\frac{\alpha_{oc}}{\delta_v}} (1 - j)^{-\frac{\alpha_{sc}}{\delta_j}} \tag{13}$$

Here it is easy to see that w hen v is small the last term becomes very small and hence shows a linear relationship between j and v. The non-linear characteristic of J–V is due to this last term of (13), where v is high (j is small). By simple algebraic manipulation (13) transforms into

$$\frac{\beta - \alpha}{1 - \alpha v - \varphi(v)} v^{\frac{\alpha_{oc}}{\delta_v}} = (1 - j)^{\frac{\alpha_{sc}}{\delta_j}} \left(1 - \frac{1}{1 - \alpha v - \varphi(v)} j\right) \tag{14}$$

For an ideal solar cell, where photocurrent is not bias dependent and shunt resistance R_{sh} is very large, the value of $1 - \alpha v - \varphi(v) = 1$. For a non-ideal case (where R_{sh} is not large and/or the photocurrent is bias dependent), it is easy to see that $1 - \alpha v - \varphi(v)$ is nearly equal to 1 while j is near to 1 or high. Hence we can state that while j is high or nearly equal to 1,

$$\left(1 - \frac{1}{1 - \alpha v - \varphi(v)} j\right) \approx (1 - j)^{\frac{1}{1 - \alpha v - \varphi(v)}} \tag{15}$$

Interestingly by approximation we can state that the previous statement (15) is also valid while j is small or near to 0. Hence for all values of j, (14) can be approximated as,

$$\frac{\beta - \alpha}{1 - \alpha v - \varphi(v)} v^{\frac{\alpha_{oc}}{\delta_v}} = (1 - j)^{\frac{\alpha_{sc}}{\delta_j} + \frac{1}{1 - \alpha v - \varphi(v)}} \tag{16}$$

For algebraic simplification considering,

$$k = \left(\frac{1 - \alpha v - \varphi(v)}{\beta - \alpha}\right)^{1/\left(\frac{\alpha_{sc}}{\delta_j} + \frac{1}{1 - \alpha v - \varphi(v)}\right)} \tag{17}$$

and

$$m = \frac{\frac{\alpha_{oc}}{\delta_v}}{\frac{\alpha_{sc}}{\delta_j} + \frac{1}{1-\alpha v-\varphi(v)}}, \tag{18}$$

we can state that (16) can be transformed into

$$v^m = (1-j)k \tag{19}$$

To express the last term of (19) in power term expansion, let us consider

$$(1-j)k = 1 - j^n \tag{20}$$

Such that

$$n = \frac{\log\{1-(1-j)k\}}{\log\{1-(1-j)\}} \tag{21}$$

By Taylor's series expansion,

$$n = k\frac{1+\frac{(1-j)}{2}k+\frac{(1-j)^2}{3}k^2+\dots}{1+\frac{(1-j)}{2}+\frac{(1-j)^2}{3}+\dots} \tag{22}$$

For an ideal solar cell, where photocurrent is not bias dependent ($\varphi(v) = 0$) and shunt resistance R_{sh} is very large, ($\alpha = 0$) the value of $k = 1$. For non-ideal case (where R_{sh} is not large and/or the photocurrent is bias dependent), it is easy to see that k is nearly equal to 1. Hence using (22), without loss of so much accuracy we can approximate the value of n as,

$$n \approx k = \left(\frac{1-\alpha v-\varphi(v)}{\beta-\alpha}\right)^{1/\left(\frac{\alpha_{sc}}{\delta_j}+\frac{1}{1-\alpha v-\varphi(v)}\right)} \tag{23}$$

Hence (19) can be approximated as (2).

Though in (18) and (22) m and n appear to be the function of j and v, it is possible to find constant effective values for m and n that are usable for all v without loss of much accuracy. Since the proposed model matches at the extremes of $v = 0$ and $v = 1$, independent of the values of δ_v and δ_j. Due to (9), it is obvious that the value of δ_v is dependent on v except for $v = 0$ and $v = 1$ and not dependent on any other physical parameters of the solar cell; therefore, any assumption of δ_v independent of v will represent a loss of accuracy, and such an assumption becomes absurd in the vicinity of $v = 0$. But since the nonlinear term vanishes for v going to 0 (see Fig. 1a), this has no severe consequences in the low-bias range, wherefore an effective value of δ_v can be used in the characteristics in the strongly nonlinear range. It would be appropriate to locate a dominant point where linear and non-linear effects, both are reflected and the dominated point lies in non-linear region. For simplification of calculation, it is interesting to see that for an ideal solar cell with infinite shunt resistance and bias independent photocurrent ($n = 1$), the tangents to (2) at $v = 0$ and $v = 1$ meet at the point where $v = (1 - m^{-1})$ (see Fig. 1a). This point can be regarded as a representative point of non-linear region and it is easy to see that the maximum portions of the J–V curve to the left and right of $(1 - m^{-1})$ can be regarded as representation of the linear and non-linear terms respectively. Hence to estimate the values of δ_v and δ_j independent of v, we can use the dominant point as $v = (1 - m^{-1})$ and using (9) and (10), the constant effective value of δ_v and δ_j can be taken as

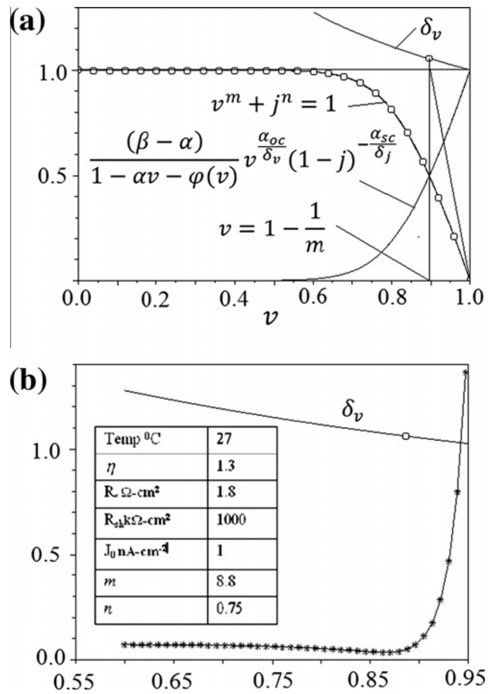

Fig. 1. (a) Normalized J–V curve of silicon solar cell (Karmalkar and Haneefa, 2008; Das, 2011) (parameters are shown in the table in (b) and the explicit model using (2). Line shows the explicit model (2) whereas the points show the implicit model (1). The tangents at the point $v = 0$ and $v = 1$ meets a point where $v = 1 - (1/m)$. The non-linear term in (13) and the variation of δ_v is also shown. (b) represents the variation of δ_v and the root-mean-square error of approximation (dotted line).

$$\delta_v = -m\log\left(1-\frac{1}{m}\right) \tag{24}$$

$$\delta_j = -\frac{m}{1-\left(1-\frac{1}{m}\right)^m}\log\left(1-\frac{1}{m}\right) \tag{25}$$

The actual variation of δ_v as a function of v is shown in the J–V range where the non-linear term dominates (Fig. 1a). The choice of the dominant point v in non-linear region for the determination of the effective value of δ_v and δ_j affects the accuracy of the approximation. For different values of v in non-linear region, δ_v and δ_j are calculated and used to calculate the root-mean-square-error of (7) and (12) as shown in Fig. 1b. It is interesting to see that the minimum error occurs at a point close to $v = (1 - m^{-1})$. Hence this point gives a satisfactory representation of a dominant point to calculate the effective values of δ_v and δ_j. To reduce the preceding expressions to simple numerical results, we use the fact that the range $4 < m < 25$ covers a very wide variety of poor-to high-quality cells. For this range of m, (24) and (25) produces $1.02 < \delta_v < 1.15$ and $1.59 < \delta_j < 1.68$. So that we can approximate,

$$\delta_v \approx 1 + \frac{0.7}{m} \tag{26}$$

$$\delta_j \approx 1.57 + \frac{0.7}{m} \tag{27}$$

Using the effective value of δ_v and the dominant point $v = (1 - m^{-1})$, we can solve (18) for v independent value of m and without loss of much accuracy the value of m can be approximated as,

$$m \approx \frac{1.57\alpha_{oc} - 0.25\alpha_{sc} - \frac{1.1}{1-\alpha}}{\alpha_{sc} + \frac{1.57}{1-\alpha}} \tag{28}$$

Using the value of m, n can be approximated as,

$$n \approx \left(\frac{1 - \alpha\left(1 - \frac{1}{m}\right)}{\beta - \alpha}\right)^{1 \Big/ \left(\frac{\alpha_{sc}}{1.57 + \frac{0.7}{m}} + \frac{1}{1 - \alpha\left(1 - \frac{1}{m}\right)}\right)} \tag{29}$$

3. Discussion

The value of m and n derived in (28) and (29) respectively are related to the physical parameters of the solar cell. This derivation gives an analytical validation of proposed model. These values are used to compute the fill factor (FF) of different solar cells using the following equation (Das, 2011):

$$FF = v_{mpp} j_{mpp} = \left(\frac{m}{n}\right)^{1/n} \left(1 + \frac{m}{n}\right)^{-\left(\frac{1}{m} + \frac{1}{n}\right)} \tag{30}$$

The derivation in this paper brings the functional relationship between the model parameters m and n and the physical parameters of the solar cell as shown in Table 1.

As shown in Table 1, when the photo-current in bias dependent solar cell like a-Si (Hegedus, 1997), the β becomes less than 1 while in other cells where the photo-current is bias independent the value of β remains 1. Similarly when the shunt resistance R_{sh} is high like Si (Das, 2011), the value of n becomes 1. While it is not large, the value of α become small but finite and hence the value of n becomes nearly equal to 1. While the usefulness and the applicability of the explicit model (2) are already demonstrated in Das (2011), Saetre et al. (2011), this paper gives an analytical basis of the explicit model showing the approximate functional relationship of model parameters and solar cell parameters.

Table 1
Physical parameters and model parameters of different solar cells.

Cell	a-Si (Hegedus, 1997)	Si (Das, 2011)	CuInSe (Das, 2011)
H	1.7	1.75	1.82
R_s (Ω-cm^2)	1.6	6.13	2.11
R_{sh} (kΩ-cm^2)	1000	3.82	0.09
J_0 (nA-cm^2)	2×10^{-2}	2	947
$J_{ph\infty}$ (mA-cm^2)	13.6	–	–
J_{phsc} (mA-cm^2)	13.3	31.6	42.0
L_c/D	20.7	–	–
V_0 (V)	0.92	–	–
V_{oc}	0.859	0.596	0.490
J_{phoc}(mA-cm^2)	9.6	–	–
(B	0.73	1	1
α_{oc}	19.59	13.16	10.41
α_{sc}	0.48	4.28	1.88
α ($\times 10^{-3}$)	0.06	4.93	129.62
M	14.33	3.16	3.96
N	1.27	1.00	1.02
FF (Reported)	0.79	0.495	0.534
FF (Calculated)	0.785	0.484	0.535

4. Conclusion

In this paper it is shown how the physically based implicit J–V equation of an illuminated solar cell, which contains the exponential, can be transformed into the recently proposed explicit model for characterization of the solar cell. The derivation is achieved by a series of algebraic manipulations, which establish the origin of the intuitive model. This paper elevates the earlier explicit model (Das, 2011; Saetre et al., 2011), which is regarded as purely empirical, to a physical model applicable to a wide variety of solar cells.

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